A coherent approach to Spacetime Foam

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A coherent superposition of N Schwarzschild wormholes is proposed as a model for spacetime foam. Following the subtraction procedure for manifolds with boundaries, we calculate by variational methods the Casimir energy. A proposal for an alternative foamy model formed by N Schwarzschild-Anti-de Sitter wormholes is here considered. Finally, a conjecture about the foam evolution is proposed.

Spacetime Foam; Black Holes; Quantum Gravity.

The term *Spacetime Foam* was used for the first time by J.A. Wheeler to indicate that spacetime may be subjected to quantum fluctuations in topology and metric at the Planck scale [1]. In a series of papers we have proposed a model for such a *foamy space* made by N coherent Schwarzschild wormholes [2, 3, 4]. The relevance of this proposal is based on a Casimir energy computation showing that quantum fluctuations of the gravitational metric shift the minimum of the effective energy from flat space (the classical minimum of the energy) to a multi-wormhole configuration. Indeed the final energy contribution to one loop is

$$\Delta_{N_w} E(M) \sim -N_w^2 \frac{V}{64\pi^2} \frac{\Lambda^4}{e}, \quad (1)$$

where V is the volume of the system, Λ is the U.V. cut-off and N_w is the wormholes number [2]. This expression shows that a non-trivial vacuum of the multi-wormhole type is favoured with respect to flat space. It is important to remark that it is the N - coherent superposition of wormholes that it is privileged with respect to flat space and not the single wormhole, because the single wormhole energy contribution has an imaginary contribution in its spectrum: a clear sign of an instability. Nevertheless the presence of an unstable mode is necessary to have transition from one vacuum (the false one) to the other one (the true vacuum) [5] . Three consequences of this multiply connected spacetime are:

1. the event horizon area of a black hole is quantized and by means of the Bekenstein-Hawking relation [6, 7], also the entropy of a black hole is quantized. In particular for a Schwarzschild black hole

$$M = \frac{\sqrt{N}}{2l_p}\sqrt{\frac{\ln 2}{\pi}}, \quad (2)$$

namely the black hole mass is quantized. Here l_p is the Planck length in natural units.

2. A cosmological constant is induced by vacuum fluctuations as shown by (1) whose value is

$$\Lambda_c = \frac{\Lambda^4 l_p^2}{N_w 8e\pi}. \tag{3}$$

When the area-entropy relation is applied to the de Sitter geometry, we obtain

$$\frac{3\pi}{\ln 2 l_p^2 N_w} = \Lambda_c. \tag{4}$$

3. Combining (3) and (4), one gets

$$\Lambda_c = \frac{\Lambda^4 l_p^2}{N_w 8e\pi} = \frac{3\pi}{\ln 2 l_p^2 N_w}. \tag{5}$$

This means that we have found a constraint on the U.V. cut-off

$$\Lambda^4 = \frac{24e\pi^2}{\ln 2 l_p^4}. \tag{6}$$

It is interesting to note that (1) can be obtained at least for $N_w = 1$ even for Schwarzschild-Anti-de Sitter wormholes (S-AdS) [8], whose line element is $ds^2 =$

$$-f(r)\, dt^2 + f(r)^{-1} dr^2 + r^2 d\Omega^2, \tag{7}$$

where

$$f(r) = \left(1 - \frac{2MG}{r} + \frac{r^2}{b^2}\right). \tag{8}$$

Λ_{AdS} is the negative cosmological constant and $b = \sqrt{-\frac{3}{\Lambda_{AdS}}}$. To consider a large N_w approach to spacetime foam even with S-AdS wormholes, one has to consider the following rescaling

$$\left\{ \begin{array}{c} R_\pm \rightarrow R_\pm / N_w \\ l_p^2 \rightarrow N_w l_p^2 \\ \Lambda_{AdS} \rightarrow \Lambda_{AdS}/N_w^2 \end{array} \right. , \tag{9}$$

where $R_\pm$ are the boundaries related to the single wormhole. This rescaling is a consequence of the boundary reduction related to the coherent superposition of wormholes wave functionals leading to the stabilization of the system This means that a selection rule has to emerge to compare the quantity

$$\Gamma_{\mathrm{N-S-AdS\ holes}} = \frac{P_{\mathrm{N-S-AdS\ holes}}}{P_{\mathrm{AdS}}} \simeq \frac{P_{\mathrm{foam}}}{P_{\mathrm{AdS}}} \tag{10}$$

with

$$\Gamma_{\mathrm{N-S\ holes}} = \frac{P_{\mathrm{N-S\ holes}}}{P_{\mathrm{flat}}} \simeq \frac{P_{\mathrm{foam}}}{P_{\mathrm{flat}}}. \tag{11}$$

In both cases, we find a non-vanishing probability that a non-trivial vacuum has to be considered. Moreover the rescaling in (9) leaves the potential (8) invariant and when N_w is very large $\Lambda_{AdS} \rightarrow 0$. This seems to suggest that not only (10) and (11) have to be compared, but it is likely that a hierarchical mechanism of the type

$$\begin{array}{c} \frac{P_{\mathrm{N-S-AdS\ holes}}}{P_{\mathrm{AdS}}} \simeq \frac{P_{\mathrm{foam}}}{P_{\mathrm{AdS}}} \\ \downarrow_{\Lambda_{AdS}\rightarrow 0} \\ Flat\quad Space \\ \downarrow \\ \frac{P_{\mathrm{N-S\ holes}}}{P_{\mathrm{flat}}} \simeq \frac{P_{\mathrm{foam}}}{P_{\mathrm{flat}}} \end{array} , \tag{12}$$

when $N_w \rightarrow \infty$. Although this picture has to be examined in detail and (12) is only at the conjecture level, an important question comes into play: why a spacetime formed by wormholes has to be preferred with respect to flat spacetime, when the last one is the one we observe. The answer that at this stage can only be conjectured is that if we consider the following expectation value on the foam state

$$\frac{\langle \Psi_F | \hat{g}_{ij} | \Psi_F \rangle}{\langle \Psi_F | \Psi_F \rangle}, \tag{13}$$

when the number of wormholes is large enough, i.e. the scale is sufficiently large, we should have to obtain

$$\frac{\langle \Psi_F | \hat{g}_{ij} | \Psi_F \rangle}{\langle \Psi_F | \Psi_F \rangle} \to \eta_{ij}, \tag{14}$$

where η_{ij} is the flat space metric. This is a test that this foamy model has to pass if phenomenological aspects have to be considered.

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References

[1] J.A. Wheeler, Ann. Phys. **2** (1957) 604; J.A. Wheeler, *Geometrodynamics*. Academic Press, New York, 1962.

[2] R. Garattini, *A Spacetime Foam approach to the cosmological constant and entropy*. To appear in Int.J.Mod.Phys. **D**; gr-qc/0003090.

[3] R. Garattini, Phys. Lett. **B 446** (1999) 135, hep-th/9811187.

[4] R. Garattini, Phys. Lett. **B 459** (1999) 461, hep-th/9906074.

[5] S. Coleman, Nucl. Phys. **B 298** (1988) 178.

[6] J. Bekenstein, Phys. Rev. **D 7** (1973) 2333.

[7] S. Hawking, Phys. Lett. **B 134** (1984) 403.

[8] R. Garattini, Class.Quant. Grav. **17** (2000) 3335, gr-qc/0006076.